

Rectangular and Circular Arrays with Independently Controlled Beamwidth and Sidelobe Level

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Abstract—A simple method for the synthesis of rectangular and circular antenna arrays with independently specified beamwidth and sidelobe level is presented. It makes use of the Taylor one-parameter design method, and thus the resulting arrays have decaying sidelobes. Knowing that the array factor of the rectangular array is the product of the array factors of two linear arrays, the rectangular array is obtained by separately designing the two constituent linear arrays, each with the desired beamwidth and sidelobe level. On the other hand, an existing method can make the transformation between a circular array and a virtual linear array. After relating the beamwidth of the circular array to that of the virtual linear array, this transformation method is used to synthesize a circular array with the wanted beamwidth and sidelobe level. Examples are given to validate the presented method for the two array types.

I. INTRODUCTION

In addition to the linear antenna array configuration, which is obtained by placing antenna elements along a line, individual radiators can be positioned in a circular ring to form a circular antenna array or in a rectangular grid to form a rectangular antenna array. Compared to linear arrays (LAs), rectangular arrays (RAs) provide more symmetrical patterns with lower sidelobes and higher gains. Circular arrays (CAs) are of very practical interest as they provide all-azimuth, i.e. 360-degree, coverage.

The pattern of a RA is the product of the array factors of two LAs in both directions of the RA [1]. Thus to design a RA with certain properties, it is sufficient to separately design the two basic LAs to reflect these properties. Uniform-spacing CA (UCA) patterns of interest can be obtained by first transforming the UCA into a virtual uniform-spacing linear array (ULA) using the technique in [2], [3], later applying a special current distribution to this virtual ULA, and then transforming it back into a UCA. The choice of the ULA excitations is responsible for the beamwidth and the sidelobe level (SLL) control of the UCA pattern.

To synthesize RAs and CAs with independently specified beamwidth and SLL, LAs with independently specified beamwidth and SLL have to be designed first, and then from the obtained distributions of these LAs, the corresponding excitations of the RA or CA under design are found. LA design for the relatively independent control of the beamwidth and SLL can be done using the modified Dolph-Chebyshev

approach presented in [4]. The disadvantage of this approach is that the resulting sidelobes have equal level, which incur more interference at and from the far out angles. Other methods for the control of LA beamwidth and SLL mostly rely on optimization algorithms like Particle Swarm Optimization, Genetic Algorithms, and other heuristic optimization algorithms.

In this paper, a simple design method for LAs with independently adjustable beamwidth and SLL is employed to synthesize rectangular and circular arrays. It builds upon the Taylor one-parameter array design [5] to produce arrays with the desired first-null beamwidth (FNBW) and SLL, where the sidelobes are decaying.

II. FORMULATION

The FNBW of a K -element Taylor one-parameter array has been derived in [6] as

$$\Theta_{FN} = \pi - 2 \arccos \left[\frac{\sqrt{B^2 + 1}}{(K-1)\frac{d}{\lambda}} \right]. \quad (1)$$

Clearly, Θ_{FN} depends on K , the value of B which determines the SLL, and on the inter-element spacing d . λ is the wavelength at the frequency of operation. The constant B is the solution of $R_0 = 4.603 \sinh(\pi B)/(\pi B)$, where R_0 is the ratio of the intensity of the mainlobe to the highest sidelobe. The value of d that results in a prescribed Θ_{FN} is given by

$$\frac{d_\Theta}{\lambda} = \frac{\sqrt{B^2 + 1}}{(K-1) \sin\left(\frac{\Theta_{FN}}{2}\right)}. \quad (2)$$

In matrix form, and for odd K (for simplicity), the array factor of this array, assumed positioned along the z -axis, is given by $\mathbf{AF} = \mathbf{I} \times \mathbf{Q} = \left[I_{-\frac{(K-1)}{2}}, \dots, I_0, \dots, I_{\frac{(K-1)}{2}} \right] \times \mathbf{Q}$. \mathbf{I} is the $1 \times K$ excitation matrix, and \mathbf{Q} is a $K \times L$ matrix, where L is the number of values in which θ has been discretized. $\mathbf{Q}_{k,l}$ is given by

$$\mathbf{Q}_{k,l} = e^{j2\pi \frac{d}{\lambda} \left(k-1 - \frac{K-1}{2} \right) [\cos(\theta_l) - \cos(\theta_0)]}, \quad (3)$$

where $k = 1 : K$, $l = 1 : L$, and $\theta_l = -\pi + 2\pi \left(\frac{l-1}{L-1} \right)$. In (3), θ_0 denotes the angle of the main lobe.

To design a linear array with independently specified SLL and FNBW having a uniform inter-element spacing d , the

value of B is first obtained, and d_Θ is calculated from (2). These serve to first design a virtual array having d_Θ as the inter-element spacing (i.e. having the desired Θ_{FN}), and the same K and SLL. The excitations for this virtual array, \mathbf{I}_Θ , are obtained from the regular Taylor one-parameter distribution. The array factor for this virtual array is $\mathbf{AF}_\Theta = \mathbf{I}_\Theta \times \mathbf{Q}_\Theta$, where \mathbf{Q}_Θ is obtained from \mathbf{Q} in (3) by replacing d with d_Θ . The excitation amplitudes of the array under design are then obtained using (4), where \mathbf{Q}^{-1} is the pseudo-inverse of \mathbf{Q} .

$$\mathbf{I} = \mathbf{I}_\Theta \times \mathbf{Q}_\Theta \times \mathbf{Q}^{-1}. \quad (4)$$

A. Rectangular Array Design

Assume that the RA under design is positioned in the $x-y$ plane parallel to the x and y axes, and that it has M elements with d_x uniform inter-element spacing along the x -direction, and N elements with d_y uniform inter-element distance along the y -direction. Desired are a SLL of R_{0x} and a FNBW of Θ_{FNx} along x , and R_{0y} and Θ_{FNy} along y . To synthesize this array, two linear arrays have to be designed. The first, assumed laid along the x -axis, has M elements with d_x uniform inter-element spacing, R_{0x} SLL and Θ_{FNx} FNBW, and the second has N elements along the y -axis with d_y uniform inter-element spacing, and R_{0y} SLL and Θ_{FNy} FNBW. The excitation amplitudes vectors of these two arrays, denoted \mathbf{I}_x and \mathbf{I}_y respectively, have to be found using the above method leading to (4). The excitation amplitudes of the RA are such that $I_{m,n} = I_{xm} \times I_{yn}$.

B. Circular Array Design

To synthesize an N_c -element circular array positioned in the $x-y$ plane with a SLL of R_{0c} and a FNBW of Θ_{FNc} , both measured in the azimuth plane, the method in [2] should be used to first obtain the number of elements in the transformed linear array. Denoting this N_l , a linear array with N_l elements, R_{0c} SLL and a FNBW equal to Θ_{FNc}/π should be designed using the above method, before making the transformation back to a circular array.

III. RESULTS AND DISCUSSION

As a first example, a 17×21 -element RA with $d_x = d_y = 0.5\lambda$ is considered. The desired SLL and FNBW are set to: $R_{0x} = -20$ dB, $R_{0y} = -25$ dB, $\Theta_{FNx} = 25^\circ$, and $\Theta_{FNy} = 35^\circ$. The main beam is steered toward $\theta_0 = 0^\circ$ and $\phi_0 = 0^\circ$. Fig. 1 shows the obtained normalized arrays factors in the $x-z$ ($\phi = 0^\circ$) and $y-z$ ($\phi = 90^\circ$). The prescribed SLL and FNBW are achieved in both planes. The slightly lower-than-desired SLLs result from the discretization of the Taylor one-parameter line source. This is expected and similar to the conventional Taylor one-parameter arrays.

Taking a 35-element UCA as a second example, the radius is selected as 1.63λ . The desired SLL in the azimuth plane is set to -25 dB and the desired FNBW in this plane is 60° . The main beam is steered toward $\theta_0 = 90^\circ$ and $\phi_0 = 0^\circ$. The transformed linear array has 33 elements, a SLL of -25 dB, and a FNBW of $60^\circ/\pi = 19.1^\circ$. The dB polar plot of

the normalized azimuth-plane array factor is shown in Fig. 2. Both the prescribed SLL and FNBW are realized. This array factor is compared to that of the same array conventionally designed for only a SLL of -25 dB, which results in a much smaller FNBW.

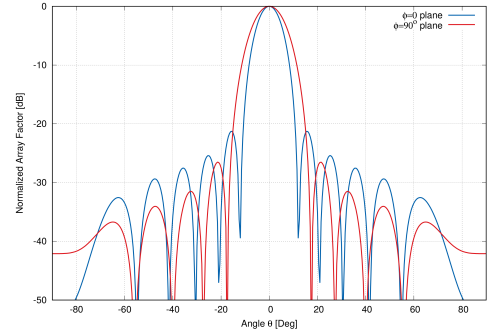


Fig. 1. Normalized array factors in the two principal plane of the example RA.

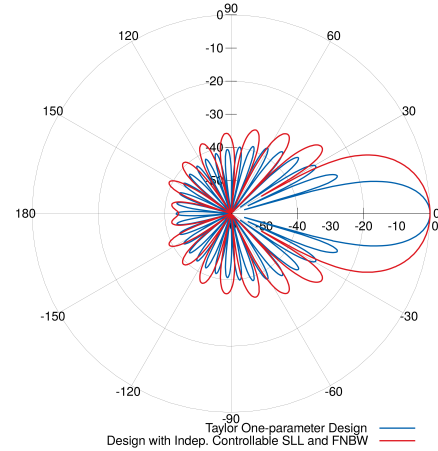


Fig. 2. Normalized array factors (in dB) of the example UCA compared to the conventional Taylor one-parameter UCA.

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