Independent Control of the Beamwidth and Sidelobe Level of Taylor One-parameter Arrays

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Abstract— A simple method for the design of linear antenna arrays having independently controllable sidelobe level and beamwidth is presented. Unlike existing methods that rely on optimization algorithms, or on a modification of the Chebyshev design method, the proposed method is based on Taylor one-parameter array design method, which is famous for providing decaying sidelobes. An equation for the first-null beamwidth of Taylor one-parameter linear arrays is first derived in terms of the array number of elements, the uniform inter-element spacing, and the prescribed sidelobe level. A virtual linear array is then designed, using Taylor one-parameter method, for the desired number of elements and sidelobe level, and for the inter-element spacing that yields the wanted first-null beamwidth. The array factor of the antenna array under design, which is probably being worked for a different inter-element spacing, is later equated to the array factor of the already synthesized virtual array. Finally, using matrix multiplication and pseudo-inversion, as shown in the presented equations, the excitations of the array under design can be obtained. These are meant for the prescribed number of elements, inter-element spacing, sidelobe level and first-null beamwidth. Examples are given to verify the correctness of the presented method.

1. INTRODUCTION

Conventional Dolph-Chebyshev arrays [1] are known to provide the narrowest beamwidth compared to other antenna array designs, but this comes at the cost of all sidelobes having the same level, which is the cause for more interference at the far out angles. A modified Dolph-Chebyshev approach is presented in [2], which enables the relatively independent control of the beamwidth and sidelobe level (SLL). Herein, the beamwidth control is translated into the enlargement of the beamwidth to a desired value from the minimum provided by the classic Chebyshev design. With this aim of adjusting the beamwidth to a larger value, there is no reason to preserve the equal-level sidelobes in the pattern, and it becomes more advantageous to have a decaying sidelobe pattern.

Other methods for the control of array beamwidth and SLL mostly rely on optimization algorithms like Particle Swarm Optimization [3], Genetic Algorithms [4], and other heuristic optimization algorithms [5].

In this paper, the focus is on linear arrays with decaying sidelobes, which are based on Taylor one-parameter design [6]. The first-null beamwidth (FNBW) of these arrays is derived in terms of the number of elements, the uniform inter-element spacing, and the SLL. For a desired FNBW, the corresponding value of the inter-element spacing is determined and is used to synthesize a virtual linear array also having the wanted number of elements and SLL. The design of this virtual array is done using the conventional Taylor one-parameter method. For the same number of elements, FNBW, SLL, and for the initially prescribed inter-element spacing of the array under design (AUD), which is probably different from the one obtained for the virtual array, the AUD elements excitations are deduced by equating its array factor to that of the virtual linear array. This involves direct matrix multiplication and matrix pseudo-inversion, without the need for any optimization.

The rest of this paper is organized as follows. Section 2 presents all the formulas needed for the design method. Examples are given in Section 3 to validate this method, in addition to comments on the results. Finally, a conclusion is given in Section 4.

2. FORMULATION

The Taylor one-parameter line-source distribution, which produces decaying sidelobes, and its corresponding space factor are reported in [7]. The line source is assumed laid along the z-axis. The space factor is given by:

$$\mathrm{SF}(\theta) = \begin{cases} l\frac{\sinh\left(\sqrt{(\pi B)^2 - u^2}\right)}{\sqrt{(\pi B)^2 - u^2}}, & u^2 < (\pi B)^2 \\ l\frac{\sin\left(\sqrt{u^2 - (\pi B)^2}\right)}{\sqrt{u^2 - (\pi B)^2}}, & u^2 > (\pi B)^2 \end{cases}, \tag{1}$$

where θ is the elevation angle, B is a constant determined from the specified SLL, $u = \pi(l/\lambda)\cos(\theta)$, and l is the length of the line source. The constant B is the solution of the following equation:

$$R_0 = 4.603\frac{\sinh(\pi B)}{\pi B}, \tag{2}$$

where R_0 is the ratio of the intensity of the mainlobe to the highest sidelobe.

A discretization of the line-source distribution yields the equation for the elements excitations of a Taylor one-parameter linear array with uniform inter-element spacing. This is given as follows:

$$a_n = J_0\left[j\pi B\sqrt{1 - \left(\frac{2n}{N-1}\right)^2}\right], \tag{3}$$

where n is the index number of each array element, going from $-(N-1)/2$ to $(N-1)/2$, N is the number of array elements (N is odd for simplicity), and J_0 is the Bessel function of the first kind of order zero. A two-dimensional version of Eq. (1), relevant to a rectangular antenna array, appears in [8].

The FNBW is derived from the line-source space factor given in Eq. (1) and is used, without loss of accuracy, for discrete Taylor one-parameter arrays. The two nulls directly next to the main lobe occur when, from the lower part of Eq. (1), $\sqrt{u^2 - (\pi B)^2} = \pi$ radians. This yields two values of θ, the right-side one being

$$\theta_{Nr} = \pi - \arccos\left[\frac{\sqrt{B^2+1}}{(N-1)\frac{d}{\lambda}}\right],$$

whereas the smaller value is

$$\theta_{Nl} = \arccos\left[\frac{\sqrt{B^2+1}}{(N-1)\frac{d}{\lambda}}\right].$$

Taking the difference $(\theta_{Nr} - \theta_{Nl})$, the following equation for the FNBW of the discrete uniform-spacing Taylor one-parameter linear array is obtained:

$$\Theta_{FN} = \pi - 2\arccos\left[\frac{\sqrt{B^2+1}}{(N-1)\frac{d}{\lambda}}\right]. \tag{4}$$

In the previous three equations, the length of the line source l is replaced by the equal length of the discrete array, or $(N-1)d$, where d is the inter-element spacing. Clearly, Θ_{FN} depends on the number of array elements N, the SLL dictated by the value of B, and on the inter-element spacing d. λ is the wavelength at the frequency of operation. A value of d can be found that results in a prescribed Θ_{FN}. This is given by

$$\frac{d_\Theta}{\lambda} = \frac{\sqrt{B^2+1}}{(N-1)\sin\left(\frac{\Theta_{FN}}{2}\right)}. \tag{5}$$

The remaining part of this section shows how to use Eq. (5) to independently control Θ_{FN} and the SLL without really having to change the inter-element spacing of the AUD.

The array factor of an N-element uniform-spacing linear array positioned along the z-axis is given by

$$AF(\theta) = \sum_{n=-\frac{(N-1)}{2}}^{\frac{(N-1)}{2}} a_n e^{jn2\pi\frac{d}{\lambda}[\cos(\theta)-\cos(\theta_0)]}, \tag{6}$$

where θ_0 denotes the angle of the main lobe. Eq. (6) is valid for odd N. In matrix form, and taking $\theta_0 = 0$ for simplicity, it can be written as

$$\mathbf{AF} = \left[a_{\frac{-(N-1)}{2}}, \cdots, a_0, \cdots, a_{\frac{(N-1)}{2}} \right] \times \mathbf{P} = \mathbf{a} \times \mathbf{P}, \tag{7}$$

where

$$\mathbf{P} = \begin{bmatrix} e^{j2\pi \frac{d}{\lambda} \frac{-(N-1)}{2} \cos(-\pi)} & \dots & e^{j2\pi \frac{d}{\lambda} \frac{-(N-1)}{2} \cos(0)} & \dots & e^{j2\pi \frac{d}{\lambda} \frac{-(N-1)}{2} \cos(\pi)} \\ \vdots & \ddots & \dots & \ddots & \vdots \\ 1 & \dots & 1 & \dots & 1 \\ \vdots & \ddots & \dots & \ddots & \vdots \\ e^{j2\pi \frac{d}{\lambda} \frac{N-1}{2} \cos(-\pi)} & \dots & e^{j2\pi \frac{d}{\lambda} \frac{N-1}{2} \cos(0)} & \dots & e^{j2\pi \frac{d}{\lambda} \frac{N-1}{2} \cos(\pi)} \end{bmatrix}. \tag{8}$$

In Eq. (7), $\mathbf{a}$ is a $1 \times N$ matrix whose elements can be obtained from Eq. (3). $\mathbf{P}$, on the other hand, is an $N \times M$ matrix, where M is the size of the vector of θ discretized between $-\pi$ and π. If the θ step size is 1 degree, $M = 361$.

To design an array with N elements, a fixed spacing d, a specified SLL, and a desired Θ_{FN}, the following steps should be taken:

1. Solve for B using Eq. (2).
2. Compute $\mathbf{a}_v$ from Eq. (3). This is the N-element excitations vector of the virtual array. This set of excitations guarantees that the virtual array has the SLL as specified. A reminder here is that the AUD and the virtual array have the same N and same B. So the AUD also has the same desired SLL.
3. Calculate d_Θ from Eq. (5). This is the inter-element spacing of the virtual array, as a result of which the virtual array has a FNBW equal to Θ_{FN}.
4. Compute $\mathbf{P}_v$ from Eq. (8) by replacing d with d_Θ. Now the array factor of the virtual array is equal to $\mathbf{a}_v \times \mathbf{P}_v$.
5. Last, since the array factors of the AUD and the virtual array are the same, the following equality holds: $\mathbf{a} \times \mathbf{P} = \mathbf{a}_v \times \mathbf{P}_v$. The excitations vector of the AUD is then given by:

$$\mathbf{a} = \mathbf{a}_v \times \mathbf{P}_v \times \mathbf{P}^{-1}. \tag{9}$$

$\mathbf{P}^{-1}$ is the pseudo-inverse of $\mathbf{P}$.

3. RESULTS AND DISCUSSION

As a first example, the AUD has the following parameters: $N = 15$, $d = 0.5\lambda$ and $\theta_0 = 90°$. The wanted SLL is -25 dB, so $B = 1.0229$. Fig. 1 shows the normalized array factor of the conventional Taylor one-parameter array, which has $\Theta_{FN} = 23.6°$, in addition to those of the arrays designed for $\Theta_{FN} = 35°$ and $\Theta_{FN} = 50°$. For the case $\Theta_{FN} = 35°$, the virtual array has $d_\Theta = 0.34\lambda$, whereas for $\Theta_{FN} = 50°$, d_Θ of the virtual array is equal to 0.242λ. The normalized values of the a_n excitations are given in Table 1 for these three cases.

As a second example, the following are taken: $N = 31$, $d = 0.5\lambda$, and $\theta_0 = 90°$. $R_0 = -35$ dB, so $B = 1.5136$. For these parameters, the conventional Taylor one-parameter array has $\Theta_{FN} = 13.9°$. For a desired $\Theta_{FN} = 45°$, the virtual array has $d_\Theta = 0.158\lambda$. For $\Theta_{FN} = 80°$, $d_\Theta = 0.094\lambda$. The normalized arrays factors for the conventional case and for $\Theta_{FN} = 45°$ and $\Theta_{FN} = 80°$ are plotted in Fig. 2.

In Figs. 1 and 2, the array factors are plotted against θ going from 0 to 180° only. This is because in the range $-180° \leq \theta \leq 0$, the array factors are just the mirror images of these plots. The results show that the achieved SLLs are slightly smaller than the prescribed levels. This is due to the approximation of the Taylor line source by a discrete array. This slight difference, which is also detected in the conventional Taylor method, becomes more negligible when N increases. Referring to Table 1, a negative a_n means an excitation that is 180° out of phase with a positive counterpart. The data in this table shows that as the desired beamwidth gets wider, the proposed design method tends to assign smaller excitations to the array's edge elements, and these excitations could become zero. This makes sense since arrays with a smaller number of elements offer a wider beamwidth. As a final note, the presented method cannot achieve a FNBW smaller than that of the conventional Taylor one-parameter, for the specified SLL. As with the modified Chebyshev method in [2], the independent control of the beamwidth means the ability to independently enlarge the beamwidth beyond the minimum achieved by the conventional method.

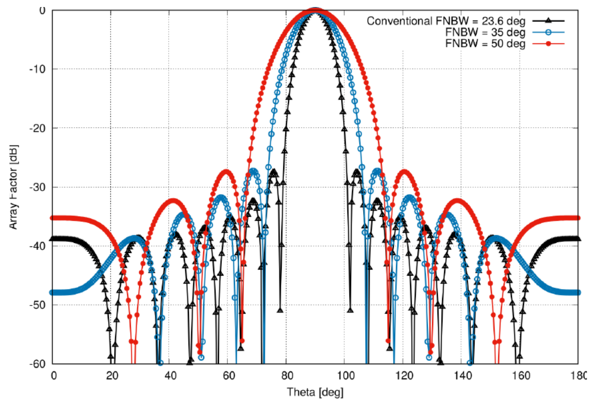

Figure 1: Normalized array factors of 15-element linear arrays all designed for -25 dB SLL: the first is a conventional Taylor one-parameter, while the second and the third are designed for 35° and 50° FNBW, respectively.

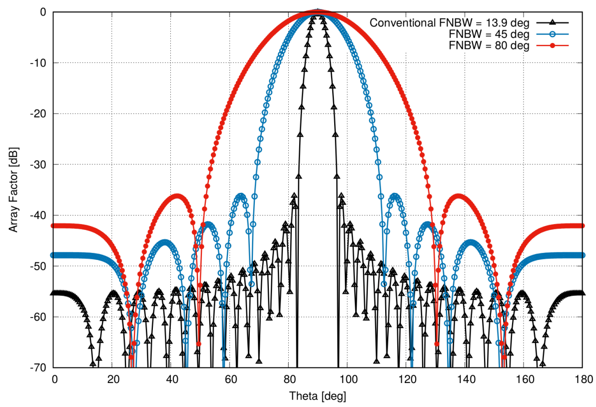

Figure 2: Normalized array factors of 31-element linear arrays all designed for -35 dB SLL: the first is a conventional Taylor one-parameter, while the second and the third are designed for 45° and 80° FNBW, respectively.

Table 1: The normalized a_ns for the arrays of Example 1.

a_n	$\Theta_{FN} = 23.6°$ (Taylor 1-param)	$\Theta_{FN} = 35°$	$\Theta_{FN} = 50°$
a_0	1	1	1
$a_{-1} = a_1$	0.973	0.934	0.886
$a_{-2} = a_2$	0.896	0.786	0.609
$a_{-3} = a_3$	0.777	0.555	0.276
$a_{-4} = a_4$	0.629	0.338	0.0124
$a_{-5} = a_5$	0.469	0.0972	-0.0012
$a_{-6} = a_6$	0.312	-0.015	0.0002
$a_{-7} = a_7$	0.172	0.009	0

4. CONCLUSION

A simple method for the design of linear antenna arrays with independently controllable sidelobe level and beamwidth was presented. The array factor was written as a matrix product. The method first synthesized a virtual array with a specifically computed inter-element spacing to obtain the desired beamwidth, and using the Taylor one-parameter method to guarantee the specified sidelobe level. A formula for the inter-element spacing in terms of the wanted beamwidth was derived for Taylor one-parameter arrays. With the array under design possibly having a different inter-element spacing, matrix multiplication and pseudo-inversion were required to obtain its excitations. This method led to array factors with decaying sidelobes, which is an advantage, and did not require the use of optimization techniques. Examples were given and were followed by a discussion.

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